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# Tax reform with endogenous borrowing limits and incomplete asset markets

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### Abstract

This paper studies different income tax reforms in an infinite horizon economy with a progressive labor income tax code, incomplete markets and endogenous borrowing constraints on asset holdings. The endogenous limits are determined at the level at which households are indifferent between defaulting and paying back their un-secured debt. The reforms we study are all revenue neutral and they eliminate capital income taxes but they differ in the changes to the labor income tax code. Our results illustrate that a successful reform has to combine the elimination of capital income taxes with an increase in the progressivity of the labor income tax code. On the one hand, this reduces the disposable income of the rich, leading to lower savings and to a lower aggregate capital. On the other hand, it allows the middle income households to save more at a higher after tax interest rate and the low income households to borrow more on a lower interest rate. This increases welfare both in the long run and throughout the transition. The welfare gains are hence obtained not through more capital accumulation but by reducing wealth and consequently consumption inequality.

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## 1. INTRODUCTION

This paper studies different tax reforms in an infinite horizon incomplete markets model with production where borrowing constraints on asset holdings are endogenous. In particular, we study the desirability of the revenue neutral elimination of capital income taxes. In our economy, capital income taxes discourage capital accumulation and in principle their elimination can lead to long run welfare gains. However, the only way to recover the lost revenue for the government is to adjust the labor income tax system. We use a flexible non-linear labor tax function which approximates well the current U.S. tax system. We study what changes in labor income taxes can lead to an increase in aggregate welfare and can gain political support.

In order to be able to study the welfare effect of a realistic tax reform, we need to have a model with a realistic wealth distribution. The fact that there is a significant proportion of individuals in debt in the data implies that a realistic model of incomplete markets should also be able to generate enough borrowing. Clearly, these two aspects are interrelated through the borrowing constraints, since they are one of the key determinants of the (equilibrium) level of debt and in general of the wealth distribution in these type of economies. In the present paper, we determine these constraints endogenously and we explicitly take this into account by calibrating the model so that the distribution of assets and the amount of debt matches the one in the data.

To endogeneize the borrowing limits, we introduce the possibility of default on financial liabilities. In particular, we assume that households can break their trading contracts every period. In this case, individual liabilities are forgiven, assets are seized, and agents are excluded from future asset trade forever. The endogenous trading limits are then set at the level at which households are indifferent between honoring their debt and defaulting.

An appealing property of endogeneizing the borrowing limits becomes more apparent when we consider policy applications such as the reforms we study. In a framework in which the equilibrium allocations exhibit imperfect risk sharing, changes in economic policy typically affect the wealth distribution. In the presence of limited commitment, these changes also affect the relative value of default and consequently the endogenous borrowing constraints. This is particularly important in models with capital accumulation, potentially, generating sizeable general equilibrium effects that interact with the borrowing limits. For this reason, in order to isolate the effect of the endogenously responding borrowing limits, we also study the case when the limits are kept unchanged after the reform.

Using the calibrated economy, we study the effect of different tax reforms quantitatively both in the long run (comparing steady states) and in the short run by analyzing the transitional dynamics. Our main result is that the only reform which has a chance to increase aggregate welfare both in the short and long run is a reform in which the elimination of capital income taxes and the subsequent increase in average labor income taxes are accompanied by an increase in the progressivity of the labor income tax system. The key is that reforms which finance the decrease in capital taxes only through an increase in the average tax rate, cannot be successful, as they put too high burden on the asset poor individuals. These agents benefit little from the higher (after tax) interest rate but lose considerably

because of higher labor taxes and a lower disposable income. A reform that increases the progressivity of the labor income tax code can therefore gain political support and increase aggregate welfare, although it taxes high income people more heavily and for that reason leads to a lower aggregate capital eventually. This implies that the reform increases welfare by decreasing inequality. In fact, the elimination of capital taxes increases the savings of the low income households, while this is more than offset by the reduction of asset accumulation of the high income individuals.

When the borrowing limits are endogenous, it is important to note that the progressive reform makes default more attractive for borrowers, as interest rates become higher. This leads to a tighter borrowing limit in the long run. In turn, this implies that the poorest households in the new steady state are richer than in the original one, an effect that contributes to a higher long run aggregate welfare. Note, however, that these last effect also shows that it is not appropriate to draw welfare calculations based upon comparing only steady states. Namely, tighter borrowing limits may hurt agents significantly in the short run, as they limit consumption smoothing potentially decreasing their welfare. However, the computed transition dynamics shows that this is not the case in our economy because the borrowing limits do not get tighter at impact but instead become slightly looser. Then they become considerably tighter very gradually over a long period of time. This implies that the reform does not immediately limit consumption smoothing for the constrained agents and it implies that aggregate welfare increases significantly not only in the long run, but also when the transition is taken into account. The reform also gains an overwhelming political support. It is supported by the asset rich, who benefit from the increase in the after-tax interest rate. Further, it is also supported by the income poor, whose tax burden did not change significantly, while they now face lower borrowing interest rates (in the short run) and higher after tax saving interest rates.

Similarly, the linear reform makes default less attractive for borrowers due to a lower interest rate. This leads to a looser borrowing limit in the long run and it implies that the poorest households are poorer than in the original steady state, contributing to a lower long run aggregate welfare. As in the previous reform, looser borrowing limits lead also to more consumption smoothing in the short run and could potentially overturn this effect during the transition. The transition, nevertheless, does not modify this picture too much. It is true that borrowing constrained agents have some additional welfare gains because of the relaxation of the borrowing constraints, but most of the low income agents would be still against the reform because it implies a significantly higher tax burden. Overall, only 25 percent will support this reform and the reform would still lead to a modest welfare loss after the transition is taken into account.

Finally, we also compare the welfare changes with the interim case in which the endogenous borrowing limits are kept at their pre-reform level. Again, we find that our qualitative findings do not change if we ignore the effect on the limits. In the linear reform, however, the immediate welfare losses would have been more severe if the limits did not adjust to the policy change. Similarly, the welfare gains from the progressive reform would have been exaggerated if the effect of the limits was not taken into account. Even though this may

sound counter-intuitive, as the limits get looser at impact, agents are forward-looking and take into account that the limits are getting significantly tighter over time.

Our work builds a bridge between two important strands of literature. First, it contributes to an increasingly growing literature in which a number of authors have introduced limited enforceability of risk-sharing contracts in models with complete markets, implicitly resulting in agent and state specific trading constraints. Among others, Kehoe and Levine (1993), Alvarez and Jermann (2000, 2001) and Krueger and Perri (2005) introduce these type of limits in exchange economies, whereas Kehoe and Perri (2002, 2004) study a production economy where investors are interpreted as countries. Since the lack of commitment leads to equilibrium allocations that exhibit imperfect risk sharing, these models are labelled endogenous incomplete market economies. However, the imperfect risk sharing result may not be robust to the introduction of capital accumulation in closed economy models. For example, Ábrahám and Cárceles-Poveda (2008) show that the equilibrium of a two agent model with endogenous production exhibits full risk sharing in the long run for standard parameterizations. Further, Krueger and Perri (2006) show that a model with a continuum of agents and endogenous incomplete markets is not able to account for the increase in US consumption inequality due to the fact that there is too much risk sharing.<sup>1</sup> Since the implications of models with full or close to full risk sharing are clearly at odds with the data, this provides a strong motivation to study limited commitment in economies with incomplete markets, where risk sharing is always limited. While the number of assets traded is still exogenous in this case, the presence of limited commitment endogenizes the amount that households can borrow. In this sense, the degree of market incompleteness becomes partially endogenous, as in the present paper.

Second, our work is related to the recent literature studying the welfare effects of capital income taxation in a context with heterogeneous agents. For example, Aiyagari (1995) studies the optimal capital income tax in a model with incomplete markets and no borrowing. In contrast to the seminal papers of Chamley (1986) and Judd (1985), who show that the optimal long run capital income tax is zero for a wide class of infinite horizon models with complete markets, the author shows that the optimal long run capital income tax is always strictly positive. Further, in a model with no borrowing and a more realistic calibration, Domeij and Heathcote (2004) find that eliminating capital income taxes in a setting with no borrowing and flat tax rates may be welfare improving in the long run, while it decreases welfare in the short run. In contrast, Davila et. al. find that the constrained efficient level of capital is much higher than the competitive equilibrium one in economies similar to us where uninsurable shocks labor income shocks. They allow fully flexible taxes and transfers. Nevertheless, we confirm the results of Domeij and Heathcote (2004) in the sense, that in our most preferred reform aggregate capital is lower than in the benchmark with our more flexible tax system. However we also show that the elimination of capital taxes can be welfare improving if it is accompanied by the increase of progressivity of the labor income tax system. Conesa and Krueger (2006, 2009) study tax reforms in the presence of a progressive labor

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<sup>1</sup>Cordoba [12] also obtains full risk sharing in a production economy with a continuum of agents, complete markets and collateral constraints.

income tax code in a setting with overlapping generations and no borrowing. In contrast, our setting is an infinite horizon economy and it allows for endogenous borrowing limits.

The rest of the paper is organized as follows. Section 2 presents the general model with incomplete markets. Section 3 presents the calibration and numerical solution of the benchmark model and Section 4 analyzes the welfare implications of a tax reform in the long run and along the transition. Finally, Section 5 summarizes and concludes.

## 2. THE MODEL

We consider an infinite horizon economy with endogenous production, idiosyncratic labor productivity shocks and sequential asset trade. The economy is populated by a government, a representative firm and a continuum (measure 1) of infinitely lived households that are indexed by  $i \in I$ .

**Households.** Households are endowed with one unit of time and they can use it to either supply labor to the firm or to consume leisure. Preferences over sequences of consumption  $c_i \equiv \{c_{it}\}_{t=0}^{\infty}$  and leisure  $1 - l_i \equiv \{1 - l_{it}\}_{t=0}^{\infty}$  are assumed to be time separable:

$$U(c_i, 1 - l_i) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, 1 - l_{it}), \quad (1)$$

where  $\beta \in (0, 1)$  is the subjective discount factor and  $E_0$  denotes the expectation conditional on information at date  $t = 0$ . We assume that the period utility function  $u : \mathbb{R}_+^2 \rightarrow \mathbb{R}$  is strictly increasing and continuously differentiable in both arguments.

Each period, household  $i \in I$  receives a stochastic labour productivity shock  $\epsilon_i$ . This shock is i.i.d. across households and it follows a Markov process with transition matrix  $\Pi(\epsilon'|\epsilon)$  and  $S_\epsilon$  possible values that are assumed to be strictly positive. A household working  $l_{it}$  hours has a pre-tax labor income of  $w_t l_{it} \epsilon_{it}$ , where  $w_t$  is the wage rate per efficiency unit of labor. Labor income taxes are assumed to be progressive and they are set by the government according to the function  $T_t^l(w_t l_{it} \epsilon_{it})$ .

To insure against their idiosyncratic labor income risk, we assume that households can trade (borrow or save) in one riskless asset, whose interest income is subject to a proportional capital income tax  $\tau_k(k_{it})$ , where  $k_{it}$  represents the beginning of period individual asset holdings. The after-tax gross return is therefore equal to  $1 + r_t(1 - \tau_k(k_{it}))$ . We assume that only savers pay taxes on interest income. Given this, capital income taxes depend on the level of assets in the following way:

$$\tau_k(k_{it}) = \begin{cases} \tau_k & \text{if } k_{it} \geq 0 \\ 0 & \text{if } k_{it} < 0 \end{cases}.$$

The households' budget constraint can be expressed as:

$$c_{it} + k_{it+1} = w_t l_{it} \epsilon_{it} - T_t^l(w_t l_{it} \epsilon_{it}) + (1 + r_t(1 - \tau_k(k_{it}))) k_{it}. \quad (2)$$

At each date, household  $i \in I$  also faces a possibly endogenous and state-dependent trade restriction on the end of period asset holdings:

$$k_{it+1} \geq \kappa_{it}$$

Throughout the paper, we assume that households cannot commit on the trading contracts and we determine the borrowing constraint  $\kappa_{it}$  endogenously at the level that prevents default in equilibrium. In case of default, we assume that individual liabilities are forgiven and households are excluded from future asset trade. Households can continue supplying labor to the firm and this implies that their only source of income from the default period is their labor income. Following Livshits, MacGee and Tertilt (2006), we also assume that there is an additional penalty  $\lambda$  that reduces after-tax labour income by  $(1 - \lambda)$  after default. This penalty can be interpreted as a reduced form for different monetary and non monetary costs of defaulting, such as the fraction of income that is garnished by creditors, the utility (stigma), the fixed monetary costs of filing, and the increased cost of consumption.<sup>2</sup>

**Production.** At each date, the representative firm uses capital  $K_t \in \mathbb{R}_+$  and labor  $L_t \in (0, 1)$  to produce a single good  $y_t \in \mathbb{R}_+$  with the constant returns to scale technology:

$$y_t = Af(K_t, L_t), \quad (3)$$

where  $A$  is a technology parameter that represents total factor productivity. The production function  $f(\cdot, \cdot) : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$  is assumed to be continuously differentiable on the interior of its domain, strictly increasing, strictly concave in  $K$  and homogeneous of degree one in  $K$  and  $L$ . Capital depreciates at the rate  $\delta$  and we denote total output including undepreciated capital by:

$$F(K_t, L_t) = Af(K_t, L_t) + (1 - \delta)K_t. \quad (4)$$

Each period, the firm rents capital and labor to maximize period profits and, in equilibrium, the two factor prices are given by:

$$w_t = f_L(K_t, L_t) \quad (5)$$

$$r_t = f_K(K_t, L_t) - \delta. \quad (6)$$

**Government and Market Clearing.** At each period  $t$ , the government consumes the amount  $G_t$  and it taxes individual labor income according to  $T_t^l(\cdot)$  and individual capital income at the rate  $\tau_k$ . The government is assumed to have a balanced budget. Let  $\Psi_t(\epsilon, k)$  be the joint distribution over individual shocks and asset holdings  $(\epsilon, k)$  and let  $k_{t+1}(\epsilon, k; \Psi)$ ,  $l_t(\epsilon, k; \Psi)$  and  $c_t(\epsilon, k; \Psi)$  be the individual asset, labor and consumption choices. The government budget constraint is equal to:

$$G_t = \int_{\epsilon, k} T_t^l [w_t l_t(\epsilon, k; \Psi) \epsilon] d\Psi_t(\epsilon, k) + r_t \tau_k \widehat{K}_t,$$

where  $\widehat{K}_t$  is the sum of positive asset holdings. Further, the labor and asset market clearing conditions require that the sum of individual labor supply times the productivity shock is

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<sup>2</sup>This punishment for default resembles the bankruptcy procedures under Chapter 7. Under this procedure, households are seized from any positive asset holdings but can keep at least part of their labour income. Whereas they are allowed to borrow after some periods, this becomes considerably more difficult and costly because their credit rating deteriorates significantly.

equal to the total labor supply, while the sum of individual capital holdings are equal to the aggregate capital stock:

$$K_{t+1} = \int_{\epsilon, k} k_{t+1}(\epsilon, k; \Psi) d\Psi_t(\epsilon, k) \text{ and } L_t = \int_{\epsilon, k} l_t(\epsilon, k; \Psi) \epsilon d\Psi_t(\epsilon, k)$$

Finally, the good's market clearing condition requires that the sum of investment and aggregate consumption, including household and government consumption, is equal to the aggregate output:

$$K_{t+1} + \int_{\epsilon, k} c_t(\epsilon, k; \Psi) d\Psi_t(\epsilon, k) + G_t = F(K_t, L_t)$$

**Recursive Competitive Equilibrium.** In the present framework, the aggregate state of the economy is given by the joint distribution  $\Psi$  of consumers over individual capital holdings  $k$  and idiosyncratic productivity status  $\epsilon$ . Further, households perceive that  $\Psi$  evolves according to:

$$\Psi' = \Gamma[\Psi],$$

where  $\Gamma$  represents the transition function from the current aggregate state into tomorrow's wealth-productivity distribution. Since the individual state vector includes the individual labour productivity and asset holdings  $(\epsilon, k)$ , the relevant state variables for a household are summarized by the vector  $(\epsilon, k; \Psi)$ .

Using this notation, the outside option or autarky value  $V$  of a household with income shock  $\epsilon$  can be expressed recursively as:

$$V(\epsilon; \Psi) = u(w(\Psi)l\epsilon - T_{\Psi}^l(w(\Psi)l\epsilon; \Psi)(1 - \lambda)) + \beta \sum_{\epsilon'} \Pi(\epsilon'|\epsilon) V(\epsilon'; \Gamma[\Psi]). \quad (7)$$

Equation (7) reflects that the autarky value is a function of the wealth-productivity distribution. Note that this is in contrast with some of the literature with complete markets and no commitment, where  $V$  is exogenous (see e.g. Alvarez and Jermann (2000, 2001)). As we will see later, this is due to the fact that the distribution determines aggregate capital accumulation, which in turn determines future wages and therefore the future value of financial autarky. On the other hand, since individual liabilities are forgiven upon default, the autarky value is not a function of the individual capital holdings. Note also that the expression in (7) implicitly assumes that the aggregate state of the economy follows the same law of motion  $\Gamma[\Psi]$  if one of the agents defaults. This is correct in the presence of a continuum of agents, since an individual deviation does not influence the aggregate variables and no one defaults in equilibrium. Note that clearing the government budget constraint requires that the labour tax function  $T^l(y; \Psi)$  depends also on the distribution.

We are now ready to define the recursive competitive equilibrium. Note, that factor prices only depend on the aggregate production factors (capital and labor) and we therefore write  $w(\Psi) = w(K, L)$  and  $r(\Psi) = r(K, L)$  in what follows.

**Definition 2.1:** Given a transition matrix  $\Pi$  and some initial distribution of shocks  $\epsilon_0 \equiv (\epsilon_{i0})_{i \in I}$  and asset holdings  $k_0 \equiv (k_{i0})_{i \in I}$ , a *recursive competitive equilibrium* relative to the capital income tax rate  $\tau_k$ , the labor income tax function  $T^l(\cdot; \Psi) : \mathbb{R}_+ \rightarrow \mathbb{R}_+^\infty$  and



government consumption  $G$ , is defined by borrowing limits  $\kappa(\epsilon; \Psi)$ , a law of motion  $\Gamma$ , a vector of factor prices  $(r, w) = (r(K, L), w(K, L))$ , value functions  $W = W(\epsilon, k; \Psi)$  and  $V = V(\epsilon; \Psi)$ , and individual policy functions  $(c, l, k') = (c(\epsilon, k; \Psi), l(\epsilon, k; \Psi), k(\epsilon, k; \Psi))$  such that:

- (i) *Utility Maximization*: For each  $i \in I$ ,  $W$  and  $(c, l, k')$  solve the following problem given  $k_0, \epsilon_0, \Pi, \Gamma, T^l$  and  $\tau_k$  and  $(r, w)$ :

$$W(\epsilon, k; \Psi) = \max_{c, k', l} \left\{ u(c, 1 - l) + \beta \sum_{\epsilon'} \Pi(\epsilon' | \epsilon) W(\epsilon', k'; \Psi') \right\} \quad (8)$$

$$\text{s.t. } c + k' = w(K, L)l\epsilon - T^l(w(K, L)l\epsilon; \Psi) + (1 + r(K, L)(1 - \tau_k(k)))k$$

$$\Psi' = \Gamma[\Psi]$$

$$c \geq 0, 0 \leq l \leq 1$$

$$k' \geq \kappa(\epsilon; \Psi).$$

- (ii) *Profit Maximization*: Factor prices satisfy the firm's optimality conditions, i.e.,  $w(K, L) = f_L(K, L)$  and  $r(K, L) = f_K(K, L) - \delta$ .

- (iii) *Balanced Budget*: The government budget constraint is satisfied, i.e.,

$$G = \int_{\epsilon, k} T^l(w(K, L)l(\epsilon, k; \Psi)\epsilon; \Psi) d\Psi(\epsilon, k) + r(K, L)\tau_k \widehat{K}, \text{ where}$$

$$\widehat{K} = \int_{\epsilon, k \geq 0} k(\epsilon, k; \Psi) d\Psi(\epsilon, k)$$

is the sum of capital holdings of those who hold non-negative assets.

- (iv) *Market Clearing*:

$$K' = \int_{\epsilon, k} k(\epsilon, k; \Psi) d\Psi(\epsilon, k) \text{ and } L = \int_{\epsilon, k} l(\epsilon, k; \Psi) \epsilon d\Psi(\epsilon, k)$$

$$\int_{\epsilon, k} c(\epsilon, k; \Psi) d\Psi(\epsilon, k) + K' + G = F(K, L)$$

- (v) *Consistency*:  $\Gamma$  is consistent with the agents' optimal decisions, in the sense that it is generated by the optimal decision rules and by the law of motion of the shock.

- (vi) *No default*:  $\underline{k}(\epsilon; \Psi)$  is the debt level such that individuals are indifferent between trading and going into autarky, i.e.,

$$\underline{k}(\epsilon; \Psi) = \{\underline{k} : W(\epsilon, \underline{k}; \Psi) = V(\epsilon; \Psi)\}. \quad (9)$$

where

$$V(\epsilon; \Psi) = \max_{c, l} \left\{ u(c, 1 - l) + \beta \sum_{\epsilon'} \Pi(\epsilon' | \epsilon) V(\epsilon'; \Psi') \right\}$$

$$\begin{aligned}
\text{s.t. } c &= \left( w(K, L)l\epsilon - T^l(w(K, L)l\epsilon; \Psi) \right) (1 - \lambda) \\
\Psi' &= \Gamma[\Psi] \\
c &\geq 0, 0 \leq l \leq 1 \\
\kappa(\epsilon; \Psi) &\equiv \sup_{\epsilon': \Pi(\epsilon'|\epsilon) > 0} \{ \underline{k}(\epsilon'; \Gamma[\Psi]) \}.
\end{aligned} \tag{10}$$

Several remarks are worth noting. First, as reflected in conditions (i) and (vi), households are only allowed to hold levels of individual capital that are above a state-dependent lower bound for each continuation state with positive probability next period. This implies that the effective limit on capital holdings  $\kappa(\epsilon; \Psi)$  faced by a household is the tightest among these state-dependent lower bounds.<sup>3</sup>

Second, the definition of the state-dependent lower bounds in (9) implies that we can think about  $\underline{k}(\epsilon; \Psi)$  as a state-dependent default threshold, since it represents the level of capital holdings such that households are indifferent between defaulting and paying back their debt. Clearly, condition (vi) implies that we only consider equilibria where the trading limits are such that default is not possible. Whereas there are many borrowing limits that prevent default in equilibrium, we consider the loosest possible ones of such limits. In other words, we study the economy with limits that are not too tight, in the sense that they satisfy (9) and (10).

As shown by Ábrahám, Á. and E. Cárceles-Poveda (2009), there exists a unique lower bound  $\underline{k}(\epsilon; \Psi)$  satisfying equation (9). Note that the existence of the default thresholds is a consequence of the fact that  $V(\epsilon; \Psi)$  is finite, while  $W(\epsilon, k; \Psi)$  goes to minus infinity as  $k$  goes to the natural borrowing limit. In addition, uniqueness simply follows from the fact that  $V(\epsilon; \Psi)$  does not depend on  $k$  while  $W(\epsilon, k; \Psi)$  is strictly increasing in  $k$ . An important implication of uniqueness is the fact that the value of staying in the trading arrangement is always higher than the autarky value if the capital holdings are above the default threshold, that is,

$$W(\epsilon, k; \Psi) \geq V(\epsilon; \Psi) = W(\epsilon, \underline{k}; \Psi) \text{ for } \forall k \geq \underline{k}(\epsilon; \Psi).$$

The fact that the thresholds are finite is a consequence of the fact that  $V(\epsilon; \Psi)$  is finite. Finally, the equilibrium default thresholds and effective limits have to be clearly non-positive. Intuitively, note that agents would not default with a positive level of asset holdings, since they could then afford a higher current consumption than in autarky and at least as high of a life-time utility as in autarky from next period on by paying back their debt.

An important property of the endogenous borrowing thresholds  $\underline{k}(\epsilon; \Psi)$  will be their dependence on the labor income shock. Ábrahám and Carceles-Poveda (2009) show theoretically that if households do not derive utility from leisure and labor taxes are flat, the higher is the productivity shock of an agent, the looser are the default thresholds, i.e.  $\frac{\partial \underline{k}(\epsilon; \Psi)}{\partial \epsilon} \leq 0$  under i.i.d. shocks. As our numerical results will show, this property is robust to the presence of a labor-leisure decision and progressive labor income taxes. Given that the ability

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<sup>3</sup>If the probability of all future shock realizations is strictly positive for any given shock, the effective limit faced by the households will not be a function of the current shock, since the trading restriction has to be satisfied for all possible continuation states. This will not be the case, however, in our calibrated example.

to borrow is a positive function of income in the data, this result is a desirable property of the present setting. Of course, a key aspect for obtaining this result is that markets are effectively incomplete.

Finally, it is important to note that the default thresholds are very closely related to the endogenous borrowing limits on Arrow securities that are defined in the literature with complete markets and limited commitment. Among others, Alvarez and Jermann (2000) and Ábrahám and Carceles (2007a) define these limits in endowment and production economies, respectively.

### 3. CALIBRATION AND SOLUTION METHOD

This section discusses the calibration for the benchmark economy as well as the solution method. Next, we study the properties of the endogenous borrowing limits, particularly the relationship between these limits and income.

**3.1. Calibration.** One of the main objectives of the calibration is that the model steady state matches the earnings and wealth distribution in the US. In addition, we target several aggregate statistics, such as the interest rate, the labor share and the investment and capital to output ratios in the US data.

The time period is assumed to be one year. For preferences, we assume the Cobb Douglas function  $u(c, 1 - l) = \frac{[c^\gamma(1-l)^{1-\gamma}]^{1-\sigma}}{1-\sigma}$ , where  $\gamma$  determines the relative importance of consumption and  $\sigma$  is the level of risk aversion. We set  $\sigma = 4$  and calibrate  $\gamma$  to match 0.42, the average labor supply of men in the Time Use Surveys of 2003-2005. This target was obtained by noting that men work 40.8 hours on average per week, while their disposable time is 97 hours after deducting sleep and ‘personal care’ ( $0.42 = 40.8/97$ ).

The production function is Cobb Douglas,  $f(K, L) = AK^\alpha L^{1-\alpha}$ , where  $\alpha = 0.36$  is chosen to match the labor share of 0.64 in the US data and the technology parameter  $A$  is normalized so that output is equal to one in the steady state of the deterministic economy. The depreciation rate is set to  $\delta = 0.08$  to match the annual investment to capital ratio in the US and the discount factor  $\beta = 0.91$  is set to match a capital to output ratio of around 3, which is the value reported for the US in Cooley and Prescott (1995). This generates an interest rate of around 4%.

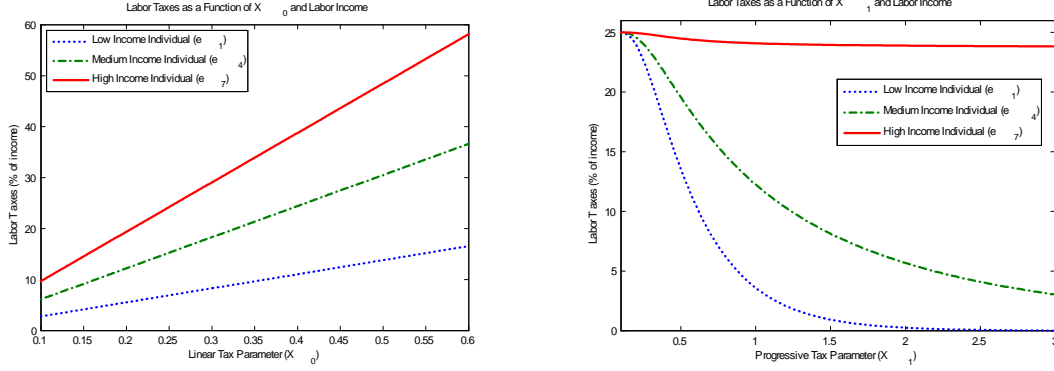
We want the income tax code to be a good approximation of the one in US. To achieve this, we assume a flat capital income tax of  $\tau_k = 0.4$ , which is very close to the value found by Domeij and Heathcote (2004) using the method of Mendoza et. al (1994). Further, we assume progressive labor income taxes. In particular, if  $y_\tau = wl\epsilon$  represents taxable income, the labor income tax is represented by the function:

$$T^l(y_\tau) = \varkappa_0 \left( y_\tau - (y_p^{-\varkappa_1} + \varkappa_2)^{-\frac{1}{\varkappa_1}} \right)$$

where  $(\varkappa_0, \varkappa_1, \varkappa_2)$  are parameters. This functional form was originally proposed by Gouveia and Strauss (1994), who estimated the function for the US income tax code. Subsequently, it has been analyzed by several authors such as Castaneda et al (1999), Smyth(2005), Conesa and Krueger (2006,2008) and Garriga and Schlagenhauf (2008). Note that, in the previous

function, the average labor income tax rate is governed by the parameter  $\varkappa_0$ , while  $\varkappa_1$  governs the degree of progressivity. In particular, when  $\varkappa_1 \rightarrow 0$ , the system becomes a flat tax, while  $\varkappa_1 > 0$  and  $\varkappa_1 < 0$  imply that the tax system is progressive and regressive respectively. To get further insights of the effects of possible tax reforms on labor income taxes, the following figure depicts the effects of changes in parameters  $\varkappa_0$  and  $\varkappa_1$  on (average) tax rates.

Figure 1: Effects of Changes in  $\varkappa_0$  and  $\varkappa_1$  on Labor Taxes



The left panel of the figure depicts the effects of the average tax rate parameter  $\varkappa_0$  for low, medium and high income individuals. As we see, the increase in  $\varkappa_0$  leads to higher labor income taxes for all the income groups, but the increase is steeper for higher income individuals due to the fact that the labor tax system is progressive. The right panel of the picture depicts the effects of an increase in the progressivity parameter  $\varkappa_1$  for the same individuals. As we see, an increase in this parameter leads to a dramatic reduction in labor taxes for lower income individuals, while the impact is much smaller for the relatively income rich. Later on, we will study the effects of particular changes in these two parameters. Gouveia and Strauss estimated this tax function for the US and they find that  $\varkappa_0 = 0.258$  and  $\varkappa_1 = 0.768$ . In the benchmark version of the model, we maintain these values and we calibrate  $\varkappa_2$  to ensure government budget balance, with a target government to output ratio of  $\frac{G}{Y} = 0.17$ . The government to output ratio is kept constant across all our experiments.

Table 1 describes the earnings process, which is a seven state Markov chain. The table displays the shock values, the stationary distribution and the transition matrix. The process, which is similar to the ones used by Diaz et. al (2003) and Davila et. al (2007), and the default penalty of  $\lambda = 0.145$  are calibrated to match the number of people in debt as well as a realistic income and wealth distribution in the benchmark steady state. In particular, the Gini coefficient for earnings is equal to 0.58, which is very close to the Gini of 0.6 in the US data, and we target the percentage of people in debt and the total financial assets held by the lowest and highest quintiles of the US wealth distribution.<sup>4</sup>

<sup>4</sup>As discussed in Livshits, MacGee and Tertilt (2006), bankruptcy filers face several types of punishment. Apart from the fact that filers cannot save or borrow, a fraction of earnings is garnished by creditors in the three year period of filing. In addition, there are utility (stigma) and fixed monetary costs of filing that imply that a fraction of consumption may be lost. To match key observations regarding the evolution of bankruptcy filings in the last decades, the authors choose a garnishment rate of 0.319 and set the other costs to zero. Given this  $\lambda = 0.145$  does not seem to be excessively high.

Table 1: Earnings Process

$\epsilon =$	0.1018	0.2192	0.5817	1.3045	2.9057	8.9879	16.0170
$\Pi^* =$	0.0996	0.2256	0.4755	0.1092	0.0545	0.0294	0.0062
$\Pi(\epsilon' \epsilon) =$	0.9400	0.0213	0.0387	0	0	0	0
	0.0265	0.8500	0.1235	0	0	0	0
	0	0.0667	0.9180	0.0153	0	0	0
	0	0	0.0666	0.8669	0.0665	0	0
	0	0	0	0.1334	0.8000	0.0666	0
	0	0	0	0	0.1235	0.8320	0.0445
	0	0	0	0	0	0.2113	0.7887

Table 2 contains information about the wealth distribution in our benchmark model and in the 2004 Survey of Consumer Finances. Since the present paper is about unsecured credit, we have tried to match some key moments of the distribution of net financial assets. In contrast, most of the macroeconomic literature focuses on the wealth distribution based on net worth, defined as the difference between total assets and total liabilities. When calculating net financial assets, we exclude the value of residential property, vehicles and direct business ownership from the assets, and the value of secured debt due to mortgages and vehicle loans from the liabilities. This level of assets represents better the amount of liquid assets that households can use to smooth out income shocks. Moreover, both residential properties and vehicles can be seen as durable consumption as much as investment.

As we see in the Table, according to the 2004 Survey of Consumer Finances, the lowest quintile of the wealth distribution, as measured by net financial assets, held -1.55% of total financial wealth, whereas 91.19 percent was held by the highest quintile. Our model matches this aspect of the distribution very well, since the assets held by the lowest and highest quintiles in the model are -1.56 and 90.35 respectively. Further, we also match reasonably well the asset holdings of the three medium quintiles in spite of the fact that they are not targeted. We also target and match well the proportion of the population in debt in the data: 24.31% (including the individuals with zero net financial assets). In this respect, our model is more reasonable than alternative models studying tax reforms in a similar framework, such as Aiyagari (1995) and Domeij and Heathcote (2004), who assume no borrowing and thus cannot capture the effect of a reform on the substantial percentage of people in debt.

Table 2: The Wealth Distribution in the Benchmark Model and in the Data

Economy	Quintiles					In debt
	Q1	Q2	Q3	Q4	Q5	%
USA (net financial assets)	-1.55	0.09	1.61	8.66	91.19	24.40
Benchmark (pre-reform)	-1.56	0.68	2.65	7.86	90.35	22.92
Gini of Earnings	US:	0.6	Benchmark:	0.58		

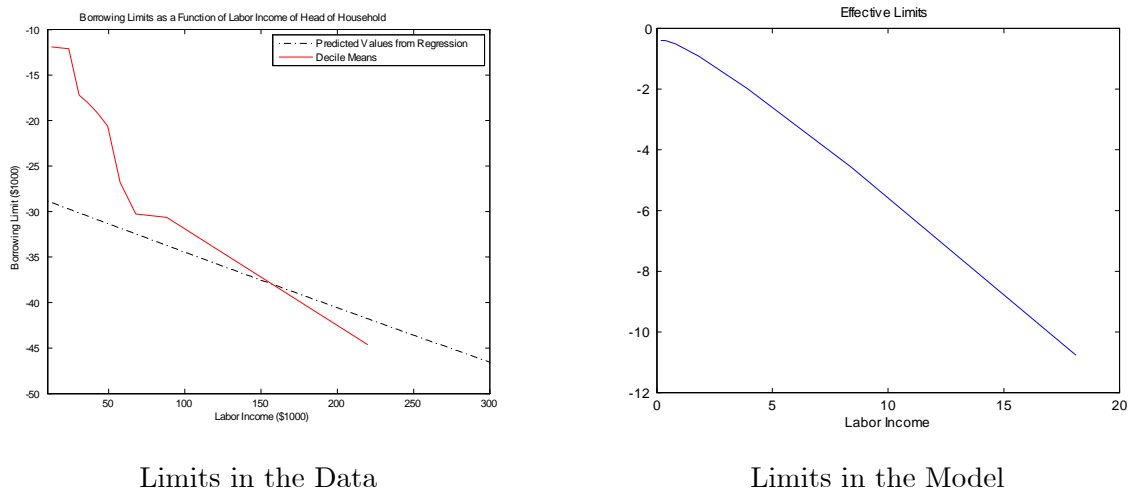
**3.2. Solution Method.** To find the solution, we use a policy function iteration algorithm that is described in detail in the Appendix. Solving the stationary distribution of the model

with endogenous trading limits involves several computational difficulties. First, our state space is endogenous, a problem that we address by incorporating an additional fixed point problem to find the state-dependent limits on the individual capital holdings. This also implies that our policy functions have to be calculated over a non-rectangular grid. Further, given that the limits in our model are endogenously determined at the level where the value function from staying in the contract is at least as large as the autarky value, it becomes clear that a good approximation of the value functions close to the limits is needed to obtain reliable results. To address this issue, we use a relatively high number of grid points, we interpolate the policy and value functions over this grid and we allow the limits to take values between grid points as well. In order to speed up the solution procedure, we update the interest rate and the borrowing limits simultaneously.

In order to evaluate the welfare effect of tax reforms, we have also computed the transition of our economy between stationary distributions due to changes in the tax code. The extra difficulty of this exercise is that not only factor prices (due to the accumulation of aggregate capital), the distribution of individuals over asset holdings and labor income change during the transition, but also the endogenous borrowing constraints. We have performed this exercise in two steps. First, we assume that the limits jump immediately to the levels of the second steady state, and compute the transition dynamics for all the other aggregate variables (factor prices and the distribution). Then, using the solution of the first step, we adjust the limits and the rest of the aggregate variables such that all the requirements of the competitive equilibrium (including the definition of the endogenous limits) are satisfied. The rationale behind this two-step procedure is that, as we will see later, the limits do not affect the transition of the aggregate variables to a large extent. So, in the second step, there are only small adjustments to be made with respect to the time path of the prices.

### 3.3. Endogenous Limits in the Benchmark Economy.

Figure 2: Endogenous Limits in the Benchmark Economy



The endogenous limits in the benchmark economy are displayed in Figure 2. The left

panel of the figure shows the level of the endogenous borrowing limits as a function of income, while the right panel plots the same variable from empirical data.

Our data source is the 2004 Survey of Consumer Finances. We only consider heads of households that are working full time and report a positive labour income and credit card limit. Our income measure is the annual labor income of the heads of households. Our income data is constructed using survey questions regarding earnings and labor supply (number of weeks worked per year).<sup>5</sup> As to the borrowing limits, the best available information is based on a question that asks the heads of households how much they can borrow on all their credit card accounts.

The left panel of Figure 1 depicts the borrowing limits as a function of labor income. Further, the right panel plots the borrowing limits as a proportion of labor income against labor income. The solid lines display data using deciles of the income distribution, taking averages within a decile. The dashed lines are the predicted borrowing limits from a regression where a third order polynomial of income, together with age, gender and education, are used to explain the limit. The figures show the predicted limits for men with the average age and educational level of the sample.

Note that the endogenous limits exhibit a similar behavior to the one in the data. In particular, they get looser with income. These findings confirm that the results in Ábrahám, Á. and E. Cárceles-Poveda (2009) are robust to the presence of endogenous labor supply and progressive labor income taxes. There, we also give further intuition behind this result.

#### 4. WELFARE EFFECTS OF TAX REFORMS

This section analyzes the welfare implications of a revenue neutral tax reform that eliminates capital income taxes at the expense of higher labor income taxes. We study two different reforms. In the first, the government replaces the loss of capital income tax revenue by increasing  $\varkappa_0$  in the labor income tax function. Note that this implies a higher average tax rate for all income levels. We label this reform as linear. In the second, the elimination of capital income taxes is accompanied by an increase in the progressivity of the labor income tax system. This is achieved by increasing  $\varkappa_1$  in the labor income tax function. Further, government budget balance also requires a higher average tax rate (through an increase in  $\varkappa_0$ ) than in the pre-reform steady state. We label this reform as progressive. Since high income agents tend to have higher asset income, the second reform has the (potential) advantage that the beneficiaries of the reform pay the costs.

**4.1. Long Run.** This section analyzes the long run welfare implications of two tax reforms. The steady state results are displayed in Table 3.

Table 3: Steady state results for the linear and progressive reforms

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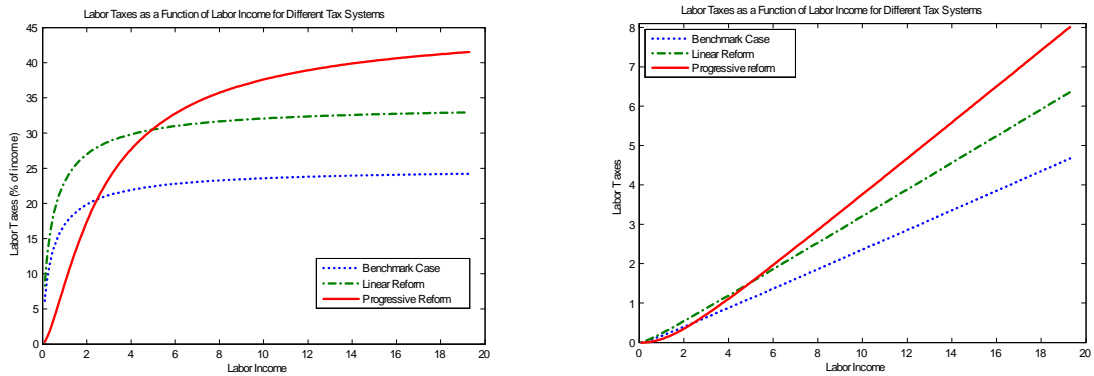
<sup>5</sup>Using alternative definitions of labor income based upon W2 forms and total household income, we obtained very similar results.

Reform	$(\varkappa_0, \varkappa_1, \varkappa_2)$	$K$	$L$	$r_\tau\%$	$r\%$	$w$	$ceq\%$	$\kappa(\epsilon_1)$
Benchmark	(0.25, 0.76, 1.37)	6.11	0.46	1.72	2.87	2.55	100	-0.656
Linear	(0.34, 0.76, 1.37)	6.00	0.44	2.72	2.72	2.57	99.38	-0.673
Progressive	(0.46, 1.53, 1.37)	5.20	0.42	3.40	3.40	2.48	106.43	-0.575

The first row of the table displays the results for the benchmark economy and the last two rows display the results for the linear and the progressive reforms respectively. Further, the different columns of the table display the labor tax function parameters  $(\varkappa_0, \varkappa_1, \varkappa_2)$ , the aggregate capital  $K$  and labor  $L$ , the saving and borrowing interest rates  $(r_\tau, r)$ , where  $r_\tau = r(1 - \tau_k)$ , the aggregate wage rate  $w$ , the (relative) aggregate welfare in consumption equivalent terms  $ceq$ , and the effective borrowing limit faced by the lowest income group  $\kappa(\epsilon_1)$ . Note that this is the most relevant borrowing limit, since it is the one faced by the lowest two income groups, who are the ones that are typically constrained. Further, in the stationary equilibrium of this model, this level of debt turns out to be the lower bound of the stationary distribution of assets.

As expected, the two reforms lead to a lower aggregate labor, reflecting the individual labor supply responses to the increasing labor income taxes. However, we observe that aggregate capital also decreases in the two reforms in spite of the increase in the after tax savings interest rate  $r_\tau$ . Note that the main reason for the reduction in aggregate savings after the tax changes is that both tax reforms reduce the net total income of the high income agents, who are the predominant savers in the economy. As we see, the increased after tax return on asset accumulation does not offset this effect. In addition, the effect is much stronger in the progressive reform, since the reduction in net income for high income individuals is considerably higher in this case. To illustrate this, we have depicted the effects of the two reforms on labor income taxes for different levels of income.

Figure 3: Effects of the Linear and Progressive Reforms on Labor Taxes



The left panel of the figure depicts the effect on labor taxes as a proportion of income ( $T^l(y)/y$ ) of the linear and progressive tax reforms for different levels of labor income, while the right panel of the figure depicts the same effects on labor taxes  $T^l(y)$ . On the one hand, the linear reform leads to higher labor income taxes for all income levels. On the other hand,



the progressive reform clearly shifts the tax burden from the income poor to the income rich by decreasing (slightly) the labor taxes for the former and increasing them for the latter. As we have seen, these effects on labor taxes reduce the net income of the rich and lead to lower aggregate savings, especially in the progressive reform.

Table 3 also reflects that eliminating capital income taxes considerably improves aggregate welfare when the progressive reform is implemented, while aggregate welfare decreases slightly with the linear reform. We can get some insight about these welfare changes by comparing factor prices and borrowing limits before and after the reform.

Consider first the progressive reform. The after tax savings interest rate  $r_\tau$  increases considerably after the reform. This benefits the savers and in particular the asset rich, for whom capital income is relatively more important. Second, the increase in progressivity benefits the relatively poor, who rely mostly on labor income and for whom the tax burden decreases after the reform. Overall, these positive effects offset the negative effect of a higher borrowing rate for the asset poor and of higher average labor income taxes for everyone (but the very poor) due to the increase in  $\kappa_0$ . Finally, the higher borrowing rate makes default more attractive and this leads to a tighter borrowing limit in the long run. In turn, this implies that the poorest households in the new steady state are richer than in the original one. This also contributes to a higher long run aggregate welfare. Note, however, that these last effect also shows that it is not appropriate to draw welfare calculations based upon comparing only steady states. Tighter borrowing limits may hurt agents in short run significantly as they limit consumption smoothing and this may decrease their welfare. The strength of the latter effect will depend on how the borrowing constraints change during the course of the transition to the new steady state and we will investigate on this later on.

Consider now the linear reform. First, while the reform leads to an increase in the after tax savings interest rate  $r_\tau$  and to a decrease in the borrowing rate  $r$ , these positive effects are offset by the fact that labor income taxes become higher for everyone. In addition to this, the table reflects that the borrowing limits become looser after the linear reform is implemented. The reason is that a decrease in the borrowing rate makes default less desirable. While looser limits might benefit borrowers in the short run, this results in the poorest agents being more indebted and therefore poorer in the new steady state, which again contributes to a lower aggregate welfare. However, looser limits (in principle) allow for more consumption smoothing and this effect can only be measured by studying explicitly the transition between the steady states.

Before doing that, we would like to emphasize one important aspect of these reforms. As suggested by the analysis of Davila et.al (2007), we expected to have welfare gains of eliminating capital income taxes. The reason is that this leads in general to a higher capital, which in turn increases the wage income of the low income and low asset agents, who primarily rely on labor income. However, this does not happen in our economy. As we see, the progressive reform is welfare enhancing but this is not due to a higher aggregate capital, which is reduced by about 15 percent, but because of a lower wealth inequality. To see this, Table 4 presents the wealth distribution in the new steady states and the number of people in debt for the two reforms. As we see, wealth inequality is lower in the two reforms but the

reduction is higher in the progressive reform. This is due to different reasons. Agents with low income and/or assets face higher (after tax) saving interest rates. Hence, they are able to save more. This is also reflected by the fact that a smaller fraction of agents ends up in debt. At the same time, high income agents are taxed more. Hence, they are able to save less. Overall, wealth becomes less concentrated both because the elimination of capital income taxes allows the poor to save more and because the accompanying rise in progressivity limits the savings of the rich.

Table 4: Wealth Distribution Before and After the Tax Reforms

Reform	Quintiles					In debt
	Q1	Q2	Q3	Q4	Q5	%
Benchmark	-1.56	0.68	2.65	7.86	90.35	22.92
Linear	-1.61	0.77	2.91	9.06	88.85	22.77
Progressive	-1.30	1.68	4.38	10.54	84.68	18.20

As explained earlier, the previous analysis does not provide with a final answer regarding the desirability of the tax reforms we consider due to the fact that we do not take into account the transition towards the new steady state. As pointed out above, the tightness of the borrowing limits can affect the degree of risk sharing in the short run and this could potentially offset (at least partially) the long run effects. To evaluate this, we have also computed the welfare changes when we take into account the transition and we discuss this next.

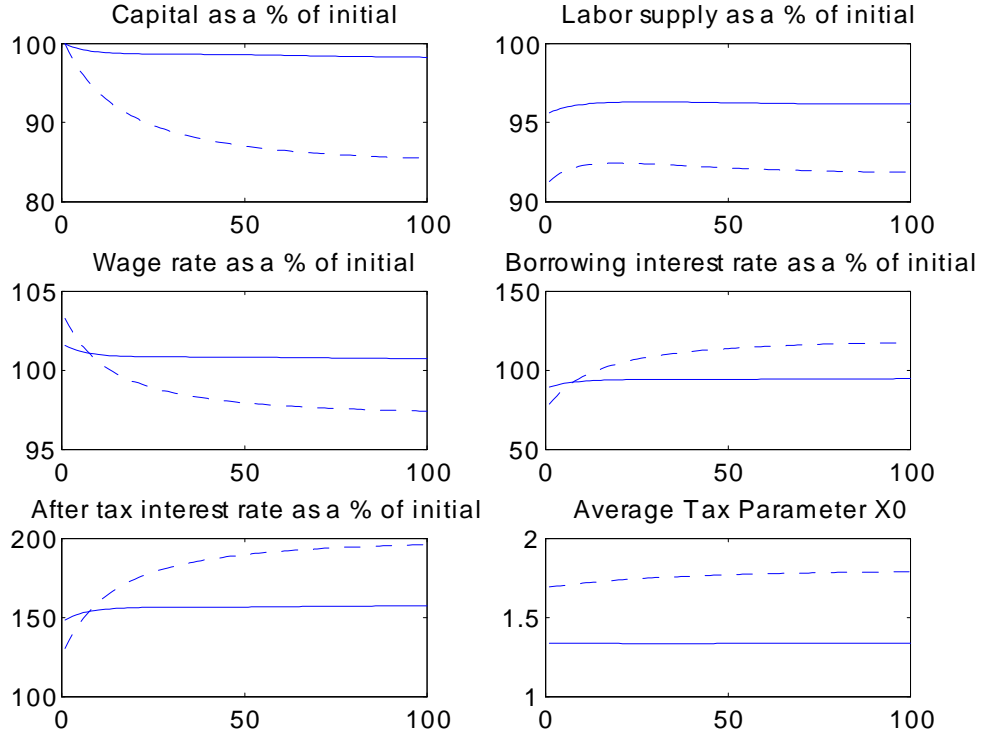
**4.2. Transition.** In what follows, we analyze the transitional effects of eliminating capital income taxes. This will help us evaluate if the long run welfare implications are offset by the short term effects along the transition. As explained earlier, the endogenous borrowing limits get looser in the linear reform and this could lead to more consumption smoothing and to a higher overall welfare. In contrast, the fact that borrowing limits get tighter in the progressive reform could lower aggregate welfare in the short run. In addition to this, studying the transition will allow us to analyze whether any of the two reforms could gain political (majority) support. At the same time, we will also be able to identify who are the winners and losers of each reform.

Note that an important difference between our framework and a fixed limit economy is that the endogenous borrowing limits are affected by the tax reform, since a change in tax policy influences the relative value of default. To disentangle the effect on the limits from the rest, we have also solved for the interim cases in which the reforms are implemented but the borrowing limits are kept the same as in the pre-reform economy. This exercise can also answer the question of what we would conclude about the welfare effects of the tax reform if we had ignored the effect of the reform on the endogenous borrowing constraints.

The transitional path for some of the key aggregate variables is displayed in Figure 4 (the dashed lines refer to the progressive while the solid lines refer to the linear reform). Several important observations emerge from the picture. First, we see that the aggregate production factors behave relatively similar in the two reforms. Second,, whereas capital decreases

gradually towards the new value in the two reforms, labor supply drops instantaneously when the reform hits. Since more progressive taxes distort labor supply more, the drop is more dramatic in the progressive reform. In this case, wages drop in the long run but they rise at impact due to the fact that labor becomes more scarce and capital is still relatively high. Similarly, the borrowing interest rate rises in the long run but it drops at impact. Third, since capital and labor are much more stable in the linear reform, the short and long run behavior of factor prices in that case is relatively similar. Finally, the last panel reflects that the average tax parameter  $\tau_0$  does not have to adjust much during the transition. This is the case especially in the linear reform, where capital and labor, hence wages experience a relatively small change.

Figure 4: Transition Path for the Aggregates in the Linear and Progressive Reforms



The movement of factor prices described above is key for understanding the time path of the effective borrowing limits  $\kappa(\epsilon_1)$  (it turns out that the lower bound of the stationary distribution is also given by  $\kappa(\epsilon_1)$  along the transition). Figure 5 depicts these limits in the two reforms. The dashed line refers to the original borrowing limit while the solid line depicts the time path of the limits throughout the transition.

As we see, the time patterns are consistent with the path of factor prices and in particular with the one of the borrowing interest rate. In the linear reform, this rate drops at impact and it increases slightly during the transition. The borrowing limit follows the same pattern as (ceteris paribus) this determines the relative value of default compared to paying back (or

rolling over) debt. Note also that looser limits are expected to be beneficial for the borrowing constrained agents, as they increase consumption smoothing. In the progressive reform, although the limits get considerably tighter in the long run, this happens very gradually. Moreover, the limits get somewhat looser at impact because the borrowing interest rate drops and this makes default less attractive. Hence, when the reform is implemented, agents are actually able to smooth consumption better. Nevertheless, they also know that they will face a slowly tightening path of borrowing constraints that will limit their future ability to smooth consumption. In other words, contrary to the arguments made in the previous subsection, this reform does not seem to limit the consumption smoothing capabilities of the constrained agents.

Figure 5: Borrowing Limits in the Linear and Progressive Reforms

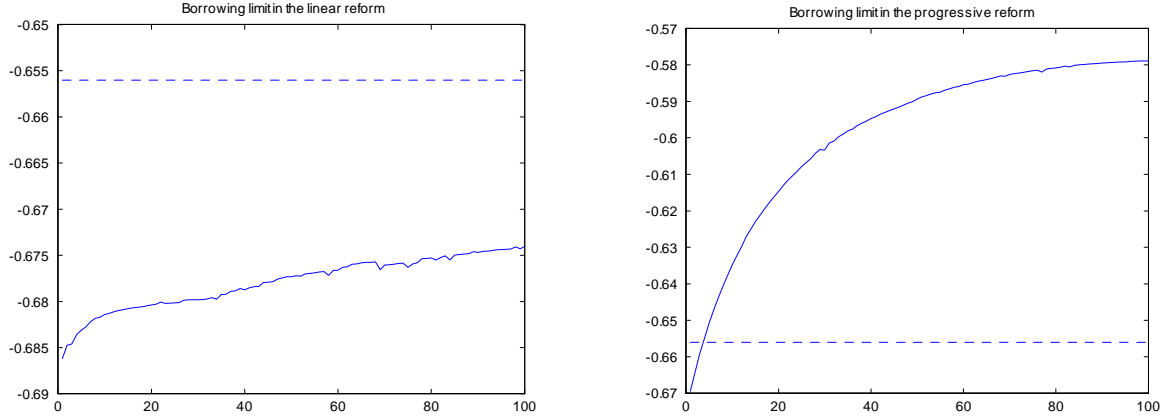


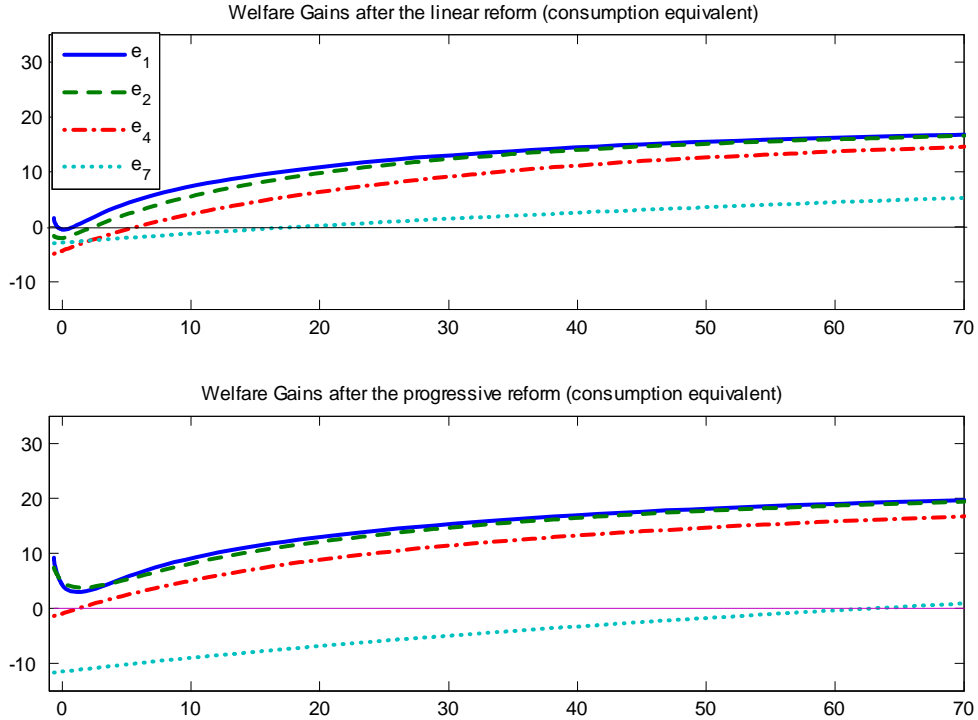
Figure 6 displays the welfare gains due to the reforms in consumption equivalent terms for individuals with different income shocks and asset levels. The upper panel of the figure reflects the welfare gains after the linear reform and the lower panel displays the gains after the progressive reform.

This figure is important for two reasons. It shows who are the agents who would be in favour and against the reform. Also, it indicates whether this reform could have public support or not. The answer to the first question is the following. For agents with positive asset position, the higher is the asset wealth of a given individual, the more this agent prefers the reform. This is not surprising, as agents with a higher asset wealth benefit more from the increase of the savings interest rate. In addition, we see that, for most asset levels, the higher is the labor income of a given individual, the less this individual will favour the reform. This is because the increased progressivity in labor income taxes will hurt this agent the most. In general, these results imply that low income individuals will tend to be in favour of the reforms, while high income individuals will tend to be against it. We also see that for low income households, the welfare gains are not monotone. In particular, we observe higher welfare gains with higher levels of debt. The reason is twofold. First, borrowing rates decrease at impact in the two reforms and this instantaneously reduces the interest burden of debt. Second, borrowing limits get looser at impact, implying that households who are

borrowing constrained can borrow more (note that this is only for a few periods in the case of the progressive reform).

Whereas the previous discussion illustrates that there are important similarities between the two reforms regarding individual welfare gains, the previous graph reflects important differences. Whereas the two lowest income groups support the progressive reform regardless of their asset levels, agents in these income groups with relatively low asset holdings or low debt do not support the linear reform. Moreover, since these agent groups constitute a relatively high proportion of the population, this results in 95% of the population supporting the progressive reform, while only 28% support the linear reform. To help explain this, note that relatively poor agents in the progressive reform end up saving and consuming more due to the lower tax burden. In other words, the progressive reform is welfare-enhancing because it reduces consumption and wealth inequality, not because it increases output. In contrast, relatively poor agents, in principle, would like to save more but have to reduce their consumption due to their increased tax burden when the linear reform is implemented. This is particularly important for poor agents and it helps explain the differences between the two reforms.

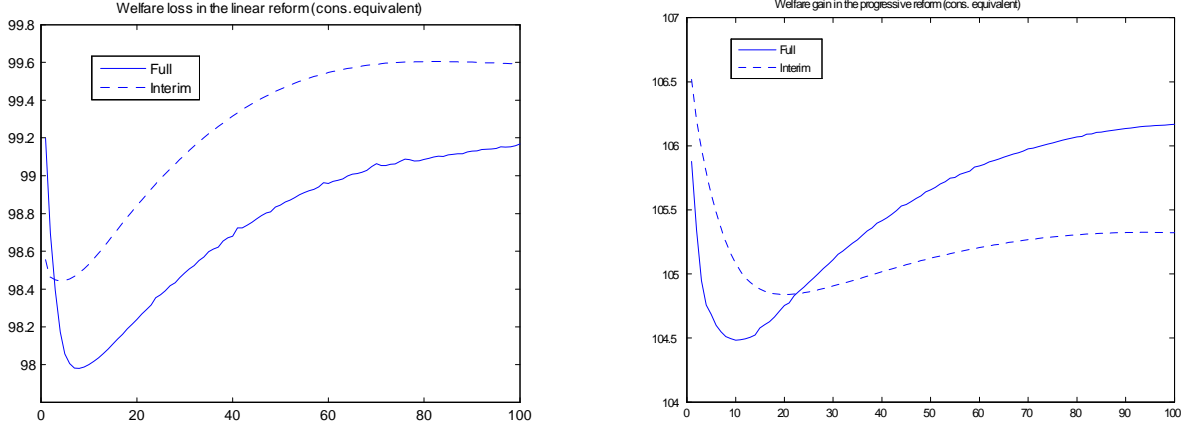
Figure 6: Welfare Gains after the Linear and Progressive Reforms



At this point, it becomes clear that we should expect that the aggregate welfare results obtained by comparing steady states do not change qualitatively if we take into account the transition. This is confirmed by Figure 7, which depicts the aggregate welfare losses for

the two reforms in consumption equivalent terms. To assess the importance of the fact that borrowing limits are endogenous, the figure also depicts the welfare changes when the limits are kept at their pre-reform level (interim reforms).

Figure 7: Aggregate Welfare Losses in the Linear and Progressive Reforms

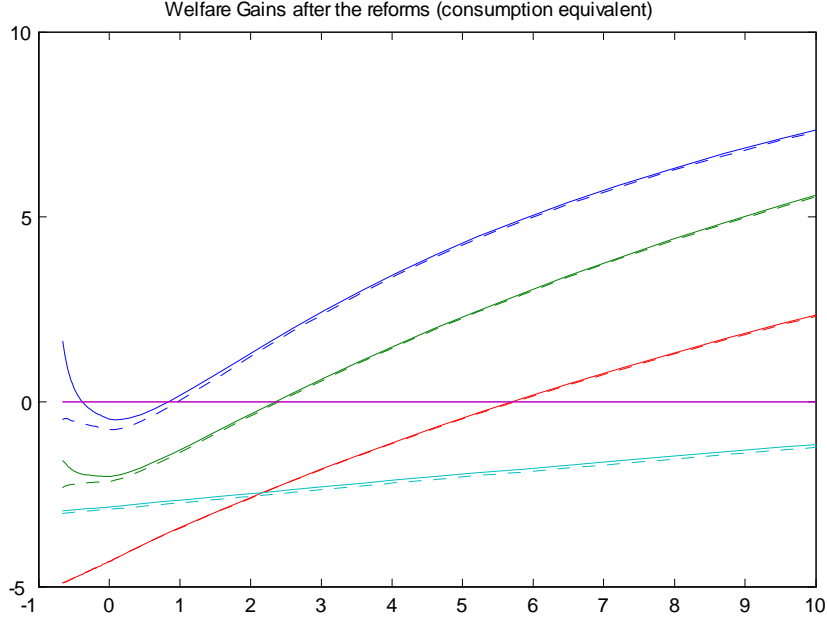


As reflected by the figure, both the quantitative and qualitative findings regarding the long run aggregate welfare changes are practically unchanged when the transition is taken into account. The linear reform implies a 0.8% welfare loss, while the progressive reform leads to a welfare gain of around 5.9%. We also see that the time path of aggregate welfare is non-monotone in both cases. To explain this, note that the fact that the limits get looser at impact leads to more consumption smoothing and to a higher welfare around the limit. Moreover, as some agents have bad draws of the income shocks and the limits get tighter, aggregate welfare starts dropping.

As stated earlier, Figure 7 also plots the time pattern of aggregate welfare for the interim reforms. Several observations are worth noting. First, we see that our qualitative findings do not change if we ignore the effect on the limits. In the linear reform, however, we see that the immediate welfare losses due to the reform would have been more severe if the limits did not adjust to the policy change. Similarly, the welfare gains from the progressive reforms would have been exaggerated if the effect of the limits was not taken into account. This may sound counter-intuitive as the limits get looser at impact in this case (see Figure 5). However, recall that agents are forward-looking and take into account that the limits are getting significantly tighter over time in this case. Otherwise, the immediate aggregate welfare gains would be higher.

To analyze the political support, Figure 8 depicts the welfare gains in consumption equivalent terms for different income groups and asset levels for the full and the interim linear reforms. As mentioned above, the impact of the change in the borrowing limits is particularly important for the relatively poor agents with negative assets, since these are the ones who are typically constrained. In particular, we see that the interim reform leads to a welfare loss for these agents, while welfare is higher for the same income groups if the limits are allowed to adjust. In fact, the political support in the interim reform is of 21% versus the support of 28% if we allow the limits to adjust.

Figure 8: Welfare Gains in the Linear Interim Reform



Finally, we find that the political support is almost unchanged when we compare the interim and the full progressive reforms, since the borrowing limits hardly change at impact. Note, however, that to obtain this result it is very important that the limits adjust gradually to the tighter level. In fact, if the limit was adjusting fully at impact, the reform would have very negative welfare consequences. In this case, agents would have a low consumption due to the fact that their next period debt would have to be considerably lower than the present one or otherwise they would have to default during the transition.

## 5. CONCLUSIONS

The present work studies whether eliminating capital taxes is desirable or not in an economy with incomplete markets, capital accumulation and the possibility of default on financial liabilities. In particular, we study competitive equilibria where the loosest possible limits that prevent default are imposed.

We first calibrate the model to match the distribution of financial assets (and unsecured) debt in the US economy. Then, we analyze the welfare implications of a revenue neutral tax reform that eliminates capital income taxes at the expense of higher labor income taxes. Our benchmark economy has a progressive labor income tax code and we study different reforms that increase the average tax rate and/or modify the progressivity of the tax system.

In our setting, we find that eliminating capital taxes can be welfare enhancing only if it is accompanied by an increase in the progressivity of the labor income tax system. In this case, aggregate capital is lower in equilibrium due to the reduction in savings of high income agents, who are actually against the reform. In contrast, low to middle income households support the reform, since their disposable income is not reduced as much and they face higher after tax saving interest rates forever and lower borrowing interest rates in the short run. It

is important to note that aggregate welfare under this reform is higher both in the long run and throughout the transition. However, the higher welfare does not arise because of a higher output but because of a reduction in inequality. In contrast to this, when the progressivity of the labor income tax code remains unchanged but average taxes are increased, the higher tax burden for all income groups more than offsets the positive effects of the reform and aggregate welfare decreases both in the long run and the short run.

Finally, we find that the endogenous borrowing limits respond significantly to tax changes. In particular, they become tighter if the labor income tax code is made more progressive, while the opposite happens when average taxes are increased but the progressivity of the labor income tax code is left unchanged. While changes in the limits affect consumption smoothing in the short run, the qualitative long run welfare effects of the two reforms remain unchanged when the transition is taken into account. However it is important to note that to obtain this result the limits have to adjust gradually. For example, if the limits become tighter at impact when the progressive reform is implemented, aggregate welfare would be dramatically lower.

Our results so far point to, at least, two interesting directions of future research. First, Davila et. al (2007) have shown that, for our calibration, constrained efficiency would require a higher aggregate capital stock than the one the competitive equilibrium can achieve. This implies that capital accumulation has to be subsidized and not taxed on average. We see that the tax system this paper uses is not flexible enough to generate welfare gains by providing incentives for higher capital accumulation. One remaining question is whether we can find a ‘simple’ tax system which can bring us closer to the constrained efficient benchmark. Our results indicate that such tax system may have to include transfers, non-linear capital income taxes/subsidies and consumption taxes. Second, our results show that changes in fiscal policy can have important impact on default incentives. This implies that if we were to allow default in our model in a way similar to Chatterjee et. al. (2007), changes of fiscal policy would have important implications on the household’s default decision and on the total amount of unsecured debt in the economy.

## APPENDIX

### Numerical Algorithm

#### 1. Computing the Stationary Competitive Equilibrium

The general algorithm used to solve for the steady state given  $\tau_k$  and a vector of parameters  $\chi = (\kappa_0, \kappa_1)$  for the labor income tax function  $T^l$  is an extension of the one in Aiyagari (1994) to endogenous limits. We use a generalized policy function iteration which relies on the first-order conditions (mainly the Euler equation) of the model. Further, we approximate all the relevant policy and value functions with linear interpolation over a finite but endogenous grid on assets. To solve the individual problem with policy iterations, we proceed as follows. Given a set of default thresholds  $\underline{k}$ , an interest rate  $r$ , a wage rate  $w$ , a tax  $\tau_k$  and a vector  $\chi$ , we let  $h$  be the vector consisting of the policy functions of interest, i.e.,  $h = [c, k', l]$ . Let  $\Lambda$  be a non-linear operator such that  $\Lambda[h, W, V; \underline{k}, r, w, \kappa_2]$  satisfies the indi-



vidual optimality conditions given  $\tau_k$ , where  $\varkappa_2$  ensures budget balance for the government. To approximate the fixed point, we follow the steps below.

Step 1: Guess an initial vector  $[h^0, W^0, V^0; \underline{k}^0, r^0, w^0, \varkappa_2^0]$ , where  $h^0 = [c^0, k'^0, l^0]$ .

Step 2: For each iteration  $n \geq 1$ , use the previous guess  $[h^{n-1}, W^{n-1}, V^{n-1}]$  and also the vector  $[\underline{k}^{n-1}, r^{n-1}, w^{n-1}, \varkappa_2^{n-1}]$  to compute the new vector  $[h^n, W^n, V^n]$  that satisfies the individual equilibrium conditions.

Step 3: Use the value functions  $[W^n, V^n]$  to find the new lower bound  $\underline{k}^n$  such that  $W^n(\epsilon, \underline{k}^n(\epsilon)) \approx V^n(\epsilon)$ , and update the grid accordingly.

Step 4: Using  $h^n$  and the distribution for the idiosyncratic shock  $\Pi$ , calculate  $\Psi^*$ , the joint (stationary) distribution of assets and income. Next, use  $\Psi^*$  to calculate the supply of capital, which is compared to the aggregate capital demanded by the firm to get  $r^n$ .

Step 5: The new parameter  $\varkappa_2^n$  of the labor income tax function is calculated given  $\Psi^*$  and  $h^n$  to satisfy the government's budget constraint.

Step 6: Repeat Steps 2-5 until convergence.

Note that our setting requires the introduction of some notable differences with respect to the standard procedure to solve models with uninsurable income shocks.

*First*, the key first-order condition is the Euler equation of the agent:

$$u'_c(c, 1-l) \geq \beta E u'_c(c', 1-l') (1+r(1-\tau_k)).$$

Recall that  $\tau_k > 0$  when agents save and  $\tau_k = 0$  when agents borrow. In this case, the agent's optimal saving policy may not be continuous (the agents optimal consumption and value function is continuous nevertheless). We first check whether there exists a strictly positive asset level satisfying the Euler equation above with equality and  $\tau_k = \tau_k$  and whether there is  $k' \in [\kappa, 0]$  satisfying  $u'_c(c, 1-l) \geq \beta E u'_c(c', 1-l') (1+r)$ . If there is a solution in the two cases, we choose the one which yields higher life-time utility. Using the equilibrium policy functions, the value functions  $W = W(\epsilon, k)$  and  $V = V(\epsilon)$  are calculated then recursively. As usual, in the above inequality for  $c'$ , we use the consumption policy functions from the previous guess.

*Second*, in Step 3, we need to update the endogenous default thresholds  $\underline{k}$  for every level of income during every iteration. Whenever  $W^n(\epsilon, \underline{k}^{n-1}(\epsilon)) > V^n(\epsilon)$ , we choose  $\underline{k}^n(\epsilon) < \underline{k}^{n-1}(\epsilon)$ . This means that we loosen the limit and this is done proportionally to  $W^n(\epsilon, \underline{k}^{n-1}(\epsilon)) - V^n(\epsilon)$ . Whenever,  $W^n(\epsilon, \underline{k}^{n-1}(\epsilon)) < V^n(\epsilon)$ , we define  $\underline{k}^n(\epsilon)$  such that  $W^n(\epsilon, \underline{k}^n(\epsilon)) = V^n(\epsilon)$ , that is, we tighten the limit.

*Third*, in Step 5, we use the parameter  $\varkappa_2$  to guarantee that the government's budget constraint is satisfied with equality period by period. This needs to be done because aggregate capital is endogenous and for any given parameter the revenue of the government depends on the wage rate, which in turns depends on aggregate capital.

*Fourth*, as opposed to the general procedure used to solve these type of economies, in which case one iterates only on  $r$  (or equivalently  $K$ ), in our procedure we have to iterate simultaneously on  $\varkappa_2$ ,  $r$  and on the default thresholds  $\underline{k}_n(\epsilon)$ . In principle, this could make the solution procedure less stable and slower. However, if one first solves the model with fixed and exogenous limits and then use the solution of such model as the initial guess for our procedure, then it converges relatively fast and without major problems.

## 2. Computing the Transition Between Steady States

When we calculate the transition between steady states we need to adjust the above procedure in the following way. First, for the sake of the exposition assume that convergence to the new steady state takes place in  $\Omega$  periods. Second, each period, assume that the government adjusts the labor income tax parameter  $\varkappa_0$ . Then we follow the steps below.

- Step 1: Guess a time series for the variables  $\{h_t^0, W_t^0, V_t^0; \underline{k}_t^0, r_t^0, \varkappa_{0,t}^0\}_{t=1}^\Omega$ , together with the time series for the distribution of individuals  $\{\Psi_t^0\}_{t=1}^\Omega$ . We then initialize the first period with stationary distribution of the first steady state ( $\Psi_1^0 = \Psi_{SS1}^*$  and  $r_1^0 = r_{SS1}$ ) and we assume that at time  $T$  we are already in the second steady state ( $\Psi_\Omega^0 = \Psi_{SS2}^*$  and  $r_\Omega^0 = r_{SS2}$ ).
- Step 2: For each iteration  $n \geq 1$  and for each time period  $1 \leq t \leq \Omega - 1$ , we use the previous guess for the next period  $[h_{t+1}^{n-1}, W_{t+1}^{n-1}, V_{t+1}^{n-1}]$  and  $[\underline{k}_{t+1}^{n-1}, r_{t+1}^{n-1}, \varkappa_{0,t}^{n-1}]$  to compute the new vector  $[h_t^n, W_t^n, V_t^n]$  that satisfies the individual equilibrium conditions.
- Step 3: Further, use the value functions  $[W_t^n, V_t^n]$  to find the new lower bound  $\underline{k}^n$  for all  $2 \leq t \leq \Omega - 1$  such that  $W_t^n(\epsilon, \underline{k}_t^n(\epsilon)) \approx V_t^n(\epsilon)$ , and update the grid accordingly.
- Step 4: Using  $h^n$  and  $\Pi$ , we calculate  $\Psi_{t+1}^n$ , the joint distribution of assets and income and then use  $\Psi_{t+1}^n$  to calculate the supply of capital  $K_{t+1}^n$ . These two variables are compared the initial guesses  $\Psi_{t+1}^{n-1}$  and  $K_{t+1}^{n-1}$  for all  $1 \leq t \leq \Omega - 1$ .
- Step 5: The new tax rate on labor for each time period  $1 \leq t \leq \Omega - 1$  is calculated given  $\Psi_t^{n-1}$  and  $h_t^n$  to satisfy the government's budget constraint.
- Step 6: Repeat Steps 2-5 until convergence in  $\{\underline{k}_{t+1}, K_{t+1}, \varkappa_{0,t}\}_{t=1}^{\Omega-1}$ .

This procedure is implemented in two steps. First we apply the procedure without Step 3, assuming that the limits are set for every period by the limits of the second steady state. This provides a very good first guess for the time path of aggregate capital, factor prices and taxes. This part of the procedure is also used to find  $\Omega$ , the endogenous length of the transition using an iterative procedure starting from  $\Omega = 3$ . Then using the solution of this first step as the initial guess, we implement the whole procedure, which involves adjusting the limits for every period.

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